Method:

The steps might seem a bit vague when written as they are below, but the 4 worked examples that follow will demonstrate exactly how the steps work. So you can skip the steps below, go straight to at the first 4 examples first and then use the steps later on as just a reminder for revision!

Always start off by write out your numbers of each category and the total

Step 1: Write out the options first in English (if there are options)

Note: if something looks different when written out in the reverse order then it is considered another valid option that we consider

For example, BW is different to WB, but BB is not different to BB so we only consider BB once

Step 2: Turn each into probability using

$$\text{probability of an event happening} = \frac{number\ of\ times\ what\ you're\ asked\ for\ can\ occur}{total\ number\ of\ options}$$

- And/then means multiply – this is for one event happening straight after the after (as long as there is no overlap. If there is we must subtract it after, but for your course there won't be so don't worry about having to sibtract)
- Or means add – this is for between your options

Step 3: Simplify

There are 3 types of questions to be aware of

Type 1: Basic probability

Type 2: Harder probability (includes with algebra)

Type 3: Putting from one bag/box to another bag/box

Note for later: Once you cover tree diagrams you will see that a lot of these questions can just be done using tree diagrams and you will see how the 'and' and 'or' knowledge links up with the tree.

Type 1: Basics

Level 1: Bronze

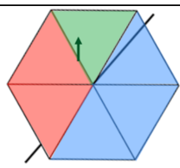
1) A fair six-sided spinner is spun two times. There are 2 red sections, 3 blue sections and 1 green section.

   i. What is the probability of getting a red twice?

   ii. What is the probability of getting a red, then blue then green

   iii. What is the probability of getting 2 different colours

Let's write out our categories and numbers for each



Key for all questions

'and' means multiply

'or' means add

2 RED

3 BLUE

1 GREEN

Total = 6

i.

Step 1: R and R

Step 2: $\frac{2}{6} \times \frac{2}{6}$

Step 3: $\frac{4}{36} = \frac{1}{9}$

ii.

Step 1: R and B and G

Step 2: $\frac{2}{6} \times \frac{3}{6} \times \frac{1}{6}$

Step 3: $\frac{6}{216} = \frac{1}{36}$

iii.

Step 1: R and B OR B and R OR R and G OR G and R OR B and G OR G and B

Step 2: $\frac{2}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{2}{6} + \frac{3}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{3}{6}$

Step 3: $\frac{6}{36} + \frac{6}{36} + \frac{2}{36} + \frac{2}{36} + \frac{3}{36} + \frac{3}{36} = \frac{22}{36}$

2) There are 6 red marbles and 5 blue marbles in a jar. What is the probability of drawing 2 red marbles and then a blue marble

i. replacement

ii. **without** replacement

6 RED

5 BLUE

Total = 11 objects

i.

Step 1:

R and R and B

Step 2:

$$\frac{6}{11} \times \frac{6}{11} \times \frac{5}{11}$$

Step 3:

$$= \frac{180}{1331}$$

ii.

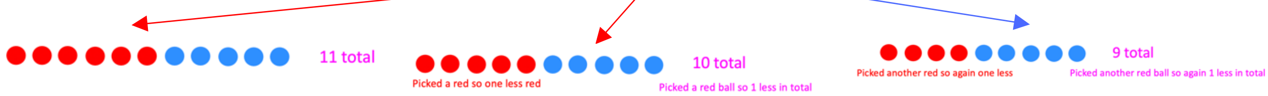
Careful. Ask yourself what is no longer in the bag since second pick will not look the same as the first pick

Step 1:

R and R and B

Step 2:

$$\frac{6}{11} \times \frac{5}{10} \times \frac{5}{9}$$



Step 3:

$$\frac{150}{990} = \frac{5}{33}$$

Level 2: Silver

3) Here are seven tiles

Jim takes at random a tile. He **does not replace** the rile. Jim then takes at random a second tile.

i. Calculate the probability that both the tiles Jim takes have the number 1 on them

ii. Calculate the probability that the number on the second tile Jim takes is greater than the number on the first tile taken

Let's colour code

2 1's

3 2's

2 3's

Total = 7 tiles

i.

Step 1:

1 and 1

Step 2:

$$\frac{2}{7} \times \frac{1}{6}$$

Step 3:

$$= \frac{2}{42} = \frac{1}{21}$$

ii.

Step 1:

We need to think of all combinations where the second number is 1 later than the first

1 and 2 OR 2 and 3

Step 2:

$$\frac{2}{7} \times \frac{3}{6} + \frac{3}{7} \times \frac{2}{6}$$

Step 3:

$$= \frac{6}{42} + \frac{6}{42} = \frac{12}{42} = \frac{2}{7}$$

4) I roll a dice twice. What is the probability that the numbers on both the dice total 5?

Step 1:

1 and 4 OR 4 and 1 OR 2 and 3 OR 3 and 2

Step 2:

$$\frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}$$

Step 3:

$$= \frac{4}{36} = \frac{1}{9}$$

5) A bag contains 10 red balls, 10 green balls and 6 white balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are of different colours?

Way 1: Basic Probability – write out all options

Here we have one event happening after the other, so we multiply
We also have options and between our options we add

Let
R represent red
G represent green
W represent white

10 Red
10 Green
6 white
Total = 26 balls

Our options are RG or GR or RW or WR or GW or WG

$$\left(\frac{10}{26}\right)\left(\frac{10}{25}\right)+\left(\frac{10}{26}\right)\left(\frac{10}{25}\right)+\left(\frac{10}{26}\right)\left(\frac{6}{25}\right)+\left(\frac{6}{26}\right)\left(\frac{10}{25}\right)+\left(\frac{10}{26}\right)\left(\frac{6}{25}\right)+\left(\frac{6}{26}\right)\left(\frac{10}{25}\right)$$

$$=\frac{100}{650}+\frac{100}{650}+\frac{60}{650}+\frac{60}{650}+\frac{60}{650}+\frac{60}{650}=\frac{44}{65}$$

Note:
We can also have done this as 2(RG)+2(RW)+2(GW)

$$2\left(\frac{10}{26}\right)\left(\frac{10}{25}\right)+2\left(\frac{10}{26}\right)\left(\frac{6}{25}\right)+2\left(\frac{10}{26}\right)\left(\frac{6}{25}\right)=\frac{44}{65}$$

Way 2: Using inclusion exclusion principle

$$\mathrm{P(different)} = 1 - \mathrm{P(same)}$$

This should make sense as all probabilities add to 1

$$1-[RR+WW+GG)$$

$$=1-\left[\left(\frac{10}{26}\right)\left(\frac{9}{25}\right)+\left(\frac{6}{26}\right)\left(\frac{5}{25}\right)+\left(\frac{10}{26}\right)\left(\frac{9}{25}\right)\right]$$

6) Here are five counters. Each counter has a number on it.

Layla puts the five counters in a bag.
She takes two counters at random from the bag without replacement.
Calculate the probability that

i. Both counters will have the number 3 on them
ii. The sum of the numbers on the two counters will be 6

1 1
3 3's
1 5
Total = 5

i.
Step 1:
3 and 3

Step 2:
$$\frac{3}{5}\times\frac{2}{4}$$

Step 3:
$$=\frac{6}{20}=\frac{3}{10}$$

ii.

Step 1:
1 and 5 OR 5 and 1 OR 3 and 3

Step 2:
$$\frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}+\frac{3}{5}\times\frac{2}{4}$$

Step 3:
$$=\frac{8}{20}=\frac{2}{5}$$